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Apparent Failure of Scaling Methods in Ramjet Connected-Pipe Testing

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The results of an evaluation of scaling methods for ramjet connected-pipe testing using previously reported data are presented. The evaluation of scaling methods includes the *pD* scaling method and another method (termed *mA* scaling here), which has not been previously documented but is in current use by the ramjet testing community. In previous experimental studies, the results have not been systematically analyzed to determine the success or failure of the scaling techniques. For the first time such an evaluation is presented, and it is concluded that neither of the two methods is successful in scaling ramjet combustors.

Introduction

RAMJET propulsion system development has received renewed attention in recent years. New ramjet systems are being developed for military and civilian use, including high-speed transportation and missile applications.[1,2] The recent resurgence of ramjets for missile applications can be attributed to an improvement in specific impulse, throttling ability, and range when compared to similar-sized solid rocket motors. Also, the United States and Germany have invested heavily in research on supersonic combustion ramjets (scramjets) toward the development of a hypersonic-speed aircraft propulsion system.

Many ramjet test programs rely on subscale combustor testing to minimize the cost during development. Methods to replicate combustion conditions in different-sized ramjet combustors (referred to as scaling in this work) have not been studied in depth. Two different methods are currently used to predict the scale effects when testing combustors that are a different size than the full-scale propulsion system.

Liquid fuel ramjets of various sizes were investigated in two experimental studies,[3,4] but the data were not analyzed systematically to evaluate the success or failure of scaling techniques. This paper presents for the first time such an evaluation. The only other data reported in the literature[5] were for a solid fuel ramjet, but neither the geometry nor the reported data are suitable for the scaling analysis presented here.

Connected-Pipe Testing

Connected-pipe testing is used for performance determination and fundamental combustion studies in ramjet and scramjet engines. A schematic of a typical connected-pipe facility is shown in Fig. 1 with the standard station numbering[2] indicated. In connected-pipe testing, the air supply is connected directly to the ramjet combustor and, therefore, connected-pipe testing considers only the combustor performance and no aerodynamic or inlet effects. By considering only the ramjet combustor, the air supply requirement and equipment necessary for testing are minimized making connected-pipe testing the most cost-effective method for evaluation and development of ramjet engines prior to freejet and flight testing.[6]

For the appropriate simulation of vehicle flight conditions, the air must be supplied at the stagnation temperatures and pressures that are to be encountered during flight. To supply the combustor with the high-temperature air necessary to simulate the conditions produced by a supersonic compression inlet, a vitiated heater is used to increase the air temperature. A vitiated heater uses combustion of a fuel added to the airflow to increase the temperature. Also, oxygen is added to the flowfield to offset the consumption of oxygen by the combustion of the vitiator fuel. There are two approaches to oxygen replenishment: 1) makeup oxygen is added to preserve the mass fraction of oxygen in the vitiated air supply and 2) makeup oxygen is added to preserve the volumetric content of oxygen in the vitiated air supply. Both of these vitiator methods are discussed in Ref. 2. By adding the fuel and makeup oxygen, the composition of the oxidizer supplied to the combustion chamber is no longer that of air because it includes combustion products from the vitiated heater. The composition can generally be considered to be all of the constituents (air, vitiator fuel, and makeup oxygen) at chemical equilibrium at the static temperature and pressure of the inlets to the ramjet combustor.

Scaling in Ramjets

In the 1960s, the Combustion Institute recognized the need to investigate combustion scaling and held a session at the 9th International Symposium on Combustion devoted to the topic of scaling with particular interest in ramjet combustors.[7,8] A method, referred to as *pL* scaling, which had been used in gas turbines, was presented as the most appropriate approach in ramjets.[8]

The method of *pL* scaling was so named because the goal of the method was to reproduce the same flowfield patterns by keeping the product of the combustor pressure p and a characteristic length L to be a constant between the scaled combustors. Currently, the method is most commonly referred to as *pD* scaling and shall be identified as such throughout this paper.

Experimental studies on ramjet scaling that are available in the open literature are limited to two studies in the mid-1970s at the U.S. Air Force Wright Laboratory[3,4] (WL) and one study at the Israel Institute of Technology in the early 1990s.[5] All of these studies used *pD* scaling as the criteria to set up the test matrix. The WL experimental programs used liquid-fueled ramjets and are discussed in detail in later sections of this paper. The study in Israel was for the development of solid-fueled ramjets, where cylindrical-shaped fuel grains were used as the inside walls of the ramjet combustor.

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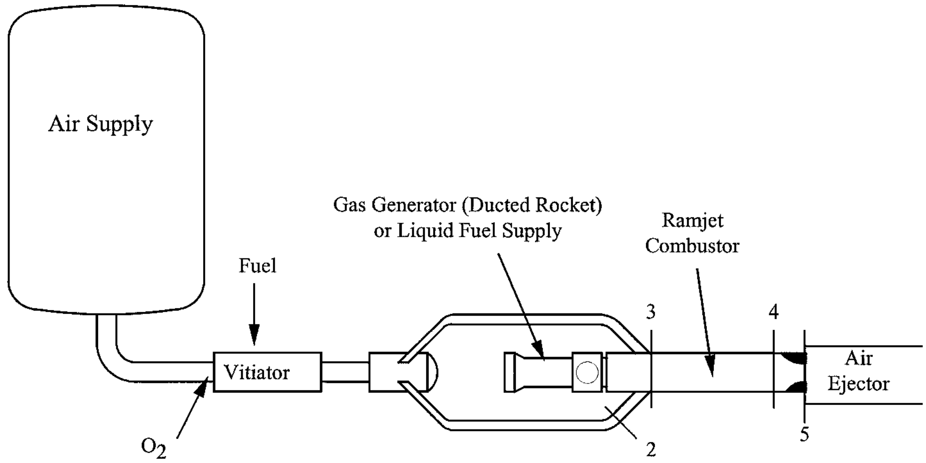

Fig. 1 Schematic of connected-pipe testing.

Whereas all of these studies concluded that *pD* scaling can be applied to ramjets, the studies lack any analysis of experimental results to support their conclusions.

There are no other methods for ramjet scaling that are presented in the open literature. However, another method (which will be termed *mA* scaling here) has been used and is possibly used with more frequency than *pD* scaling in the ramjet testing community. The method of *mA* scaling is an attempt to produce the same pressure in the subscale ramjet combustor as in the full-scale combustor. In practice the same pressure is only achieved if the thermal efficiency of the combustor is the same between the full-scale and subscale combustors. A review of the analytical reasoning of both *pD* and *mA* scaling is discussed in the following section.

Similarity Considerations in Ramjet Combustion

Absolute similarity between different-sized combustors implies that the combustion process occurs exactly the same. However, as stated by Spalding,[7] it is not physically possible to replicate the same combustion process exactly in different-sized combustors. Therefore, in combustion modeling a partial similarity approach is taken because a strict adherence to similarity theory cannot be maintained in all processes.[9]

Geometry

Geometric similarity is a first step in the attempt to achieve similarity in different-sized combustors. The desire to have the same geometry for the combustor as well as the fuel injection system is because of geometry effects on replicating the same fluid mechanics flow structure. Geometric similarity of a subscale combustor and a full-scale combustor is achieved when all length dimensions in all coordinates have the same linear-scale ratio.[10] That is, the ratio of each scaled length to the full-scale length is constant. In geometric similarity, all angles and all flow directions are preserved.

The result of observing geometric similarity in typical ramjet combustors is constant values for some nondimensional groupings of parameters. The parameters include the ratio of combustor length to combustor diameter L/D, combustor to nozzle area ratio A_4/A_5, and the ratio of inlet area to combustor area A_2/A_4. The nozzle area ratio A_4/A_5 is held constant to have the same Mach number in the combustor. The inlet area ratio A_2/A_4 is matched so that the sudden expansion region of the air inlet is reproduced to try to replicate the recirculation flowfield at the dump station.

A parameter that is not consistent with geometric similarity, but is sometimes matched in scaling, is the characteristic length L^*. The characteristic length is defined as

$$L^* = (A_4/A_5)\, L \tag{1}$$

where L is the combustor length. L^* has the dimensions of length and is used to replicate particle residence time in certain conditions. It should be noted that L/D, A_4/A_5 , and L^* cannot all be matched at the same time. When the nozzle area ratio A_4/A_5 is matched, matching L^* is reduced to simply matching the same overall length from the full-scale to the subscale.

Mixing

Ramjet combustors are characterized by a turbulent diffusion flame. Mixing between the fuel and air is promoted in recirculation zones in the combustor, where the flame holding generally occurs. In coaxial ramjets and in side-dump ramjets, the recirculation zone is caused by sudden expansion from the air inlet to the combustor.

In considering similarity of mixing and transport processes, similarity parameters from viscous flow and diffusion relations are considered. Dynamic similarity of turbulent mixing can be achieved by maintaining a constant Reynolds number,

$$Re = \rho V D/\mu \tag{2}$$

where ρ is the density, V is velocity, D is a characteristic length dimension, and μ is the viscosity.

Other fluid flow similarity parameters include Froude number Fr and Grashof number Gr, both of which consider bouyancy effects in the flow. Bouyancy effects in high Reynolds number flows such as in ramjets are not considered important in combustion modeling.[9] Also, because ramjets are governed by turbulent mixing, Schmidt number, which is a measure of the molecular mixing, may be considered unimportant in ramjet combustion. Therefore, Reynolds number matching is considered most important for similarity in the mixing and transport processes between different-sized combustors. However, there have been studies that indicate at high values of Reynolds number, such as those in ramjets, the flow patterns may become independent of Reynolds number.[9] Fuel distribution and mixing for the complex geometry of solid-fuel ramjets are covered thoroughly in Refs. 5 and 11.

Compressibility

Similarity of compressibility is achieved by matching Mach number of the flow. Mach number controls the ratio of dynamic to static pressure and affects total pressure loss and flame holding in the combustor. Compressibility effects in most ramjet combusting flowfields are considered to be slight. In fact, even though compressibility effects are considered to become important in most flows around Mach 0.3 and greater, one study states that compressibility effects in ramjet combusting flowfields are not significant until Mach 0.8 and higher.[7]

Heat Transfer

Usually, radiation heat transfer is not addressed in combustion similarity analysis and is sometimes considered insignificant in ramjets.[5] Other studies have noted that modeling radiation effects cannot be done in different-sized combustors of constant Reynolds number.[9] Also, because the flow in ramjets is turbulent, molecular conduction should have only minor effects and is not usually considered in similarity of heat transfer effects.

The requirement of similarity in convective heat transfer is met by matching Nusselt number, which is a function of Reynolds number and Prandtl number. If the different-sized combustors have the same fuel at the same thermal efficiency, the Prandtl number is constant. Therefore, by matching the Reynolds number, the convective heat transfer similarity requirements are maintained.

In combustion with two phase mixtures, several additional parameters must be maintained to assure similarity between the subscale and full-scale combustor. These parameters involve the atomization of fuel and thermal and diffusional interactions between phases. As noted in previous works,[12] the quantities that should be maintained are the ratio of fuel velocity to gas velocity, the ratio of ignition delay to residence time, and the ratio of evaporation time to residence time.

Chemical Kinetics

Chemical kinetic relations are associated with several important processes in ramjet combustion and include the ignition time and reaction time, which are the primary concerns in providing similar chemical kinetics in ramjets. The nondimensional similarity parameters used for chemical kinetics are the ratio of residence time to reaction time and the ratio of residence time to ignition time.

The ratio of the residence time to the reaction time is the so-called Damköhler's turbulent mixing number (also referred to as Damköhler's first number), which can be expressed as

$$\mathcal{D} = \frac{\tau_{\text{res}}}{\tau_{\text{react}}} \tag{3}$$

where τ_{react} is the reaction time and τ_{res} is the particle residence time, which can be expressed as

$$\tau_{\text{res}} = L/V \tag{4}$$

where L is the combustor length and V is the velocity in the combustor. Previous studies[7–9] have shown the relationship between reaction time and combustor pressure p to be

$$\tau_{\text{react}} = 1/p^{n-1} \tag{5}$$

where n is the reaction order. In the combining of Eqs. (3–5),

$$\mathcal{D} = \frac{p^{n-1}L}{V} \tag{6}$$

The value of n is usually assumed to be 2, which corresponds to a bimolecular reaction. Some studies have used experimental data curve fits to find a value for n in air breathing engines other than ramjets. The values for n in these investigations[10] had values close to 2.

Now, consider the ratio of residence time to ignition time to be constant,

$$\tau_{\text{res}}/\tau_{\text{ig}} = \text{const} \tag{7}$$

Previous studies[5] have shown the relation between the ignition time τ_{ig} and pressure to be

$$\tau_{\text{ig}} \propto 1/P \tag{8}$$

Now, if the ratio of the residence time to the ignition time is held constant in scaling, as shown in Eq. (7), the resultant relation from Eqs. (8) and (4) is

$$pL/V = \text{const} \tag{9}$$

It can be seen that the conditions of constant Damköhler number $\mathcal{D}$ in Eqs. (6) and (9) are simultaneously satisfied only for the case of $n = 2$.

Other parameters that should be considered for chemical kinetic similarity in different-sized ramjet combustors are the inlet enthalpies, fuel and air composition, and the equivalence ratio. A common inlet temperature of both air and fuel is used to match the inlet enthalpy closely.

Scaling Methods

There are two methods used for ramjet combustion scaling in connected-pipe testing. These are based on different assumptions and ways to consider matching the various similarity parameters associated with mixing and chemical kinetics.

pD Scaling

The method referred to as pD scaling is based on using different combustor pressures to match the Reynolds number and matching trends in the chemical kinetic similarity parameters. Geometric similarity is maintained between the full-scale and subscale combustors. Considering the combustor length to diameter L/D to be constant, the chemical kinetic similarity requirements of Eqs. (6) and (9) can be expressed in terms of the combustor diameter D, instead of combustor length L, as

$$pD/V = \text{const} \tag{10}$$

where the reaction-order exponent n in Eq. (6) is assumed to be 2.

For similarity in mixing, the Reynolds number is matched. The assumption is made that the flowfield can be treated as an ideal gas and, therefore, Reynolds number matching can be expressed as

$$Re = \frac{\rho VD}{\mu} = \frac{(p/RT)VD}{\mu} = \text{const} \tag{11}$$

which simplifies to

$$pVD = \text{const} \tag{12}$$

because the temperature, gas constant and viscosity of the flow are matched for an ideally scaled combustion condition.

The pD scaling approach is based on the assumption that the velocities are very similar in the different-sized combustors leaving the product of combustor pressure and diameter to be constant. If the scaling method works so that the thermal efficiency is the same in both combustors, the temperature of the combusting flow will be very similar for the two combustors, even though a difference in combustion pressure exists. This is true because temperature is a weak function of pressure in ramjets. The similar temperature values result in similar speeds of sound values. Because the Mach number is matched for compressibility effects, the velocities in the different-sized combustors are similar. Again, this condition is met only if the thermal efficiency values are sufficiently close between the combustors.

In practice, the experimenter cannot fix pD to be constant when performing experiments because the combustion pressure is a measure of the combustion performance. The pD scaling is attempted by sizing the mass flow through the combustor according to the change in diameter as

$$\dot{m}/D_4 = \text{const} \tag{13}$$

If the method provides appropriate similarity between the different-sized combustors, the product of the combustor pressure and the diameter pD is the same in the different-sized combustors, and the thermal efficiencies of the two combustors are the same.

mA Scaling

Another scaling method used by several facilities has not been presented in the open literature. The method is based on matching the combustor pressures of the full-scale and subscale combustors. This method, referred to as mA scaling in this work, is usually applied with geometric similarity. Occasionally, the method is applied with a constant L^* resulting in the same combustor length for the different diameter combustors. Using this method, the chemical kinetic parameters of reaction time and ignition time are matched exactly if the method is successful (produces the same pressure in the full and subscale combustors). When L^* is matched, the purpose is to also match the particle residence time τ_{res} in addition to the ignition and reaction times.

The choice to use a constant L/D or a constant L^* depends on the speed of the flow in the combustor. If the residence time is significantly greater than the reaction time, L/D is usually held constant. If the reaction time is sufficiently close to the residence time, the length of the different diameter combustors is usually held constant (constant L^*). The logic is that if the particle is in the combustor much more time than it takes to burn, appropriate scaling should be more related to mixing, which should occur in shorter distances in the subscale combustor. However, for situations such as those encountered in scramjet (supersonic ramjet) combustion, the flowfield is sufficiently fast so that the residence time becomes the most important parameter to match.

For matching the mixing conditions, the method assumes that the recirculation patterns are produced similarly in the different-sized combustors despite the Reynolds number not being matched. Previous studies[9] on combustion modeling have noted that the flowfield in these types of combustors may be Reynolds number independent.

In practice, this method for scaling is attempted when the mass flow rates are scaled according to the area ratios of the combustors as

$$\dot{m}/A_4 = \text{const} \tag{14}$$

In this paper, this method is referred to as *mA* scaling because of the method of application (mass flow is scaled with area, not diameter as in *pD* scaling). Using this method, if the thermal efficiency of the different-sized combustors is matched, the pressure in the different-sized combustors is also matched (the goal of this method).

Evaluation of Scaling Methods

Three previous experimental programs[3–5] that investigated scale effects in ramjet combustion have been presented in open literature. Two were performed using liquid-fuel ramjets during the mid-1970s at WL.[3,4] The third study was performed using solid-fuel ramjets at Technion, the Israel Institute of Technology, in the early 1990s.[5] The effectiveness of the *pD* and *mA* scaling methods is evaluated here using the data from the WL programs. The WL data have never been analyzed in a systematic manner such as that used here. The Technion data are not analyzed in this study for reasons discussed next.

The most recent study of ramjet scaling was performed at Technion using a solid-fuel ramjet.[5] The test program did not consider different-sized combustors, but rather fuel regression in a single solid-fuel ramjet. In this ramjet geometry, the combustor wall diameter changes throughout a single test as the solid fuel is consumed. Therefore, the experimental results at different times for a single combustor were used to consider combustion conditions at different diameters. The tests were designed based on the *pD* scaling method, which the authors conclude as applicable to solid-fuel ramjets based on the experimental results that were reported.[5] However, the authors chose to use combustion efficiency based on characteristic exhaust velocity C^* as the performance parameter to consider for scaling. This method is identical to the parameter η_{C^*} that is discussed in Refs. 12 and 13. As shown in Refs. 12 and 13, combustion efficiency based on C^* is not suitable for use in characterizing the efficiency of the combustion process in ramjet combustors. This issue is discussed further in the "Experimental Results" section. Because no measurement data from that test program were reported, further analyses of the experimental results were not possible in this study.

The emphasis of the WL test programs was to consider the influence of many geometric parameters on thermal efficiency in different-sized ramjet combustors. In all, several hundred connected-pipe tests were performed. The test conditions for the different-sized combustors included tests that matched the criteria for both *pD* and *mA* scaling methods. Although the WL test programs used *pD* scaling for test planning of certain test conditions, the primary focus of the tests was not the investigation of scale effects. Consequently, no conclusions applicable to the scaling methods were presented. In fact, the test conditions that match the *mA* scaling method were not planned on the basis of *mA* scaling. Fortunately, the technical reports of these test programs[3,4] presented much of the measurement data so that more analysis could be performed as described in the following sections.

Experimental Approach

The WL experimental work consisted of two different test programs. The first test program included 2-, 3-, 4-, and 5-in.-diam liquid-fueled ramjet combustors, was reported[3] in 1974, and is referred to as WL74 in this work. The second test program consisted of 5-, 8-, and 12-in.-diam liquid-fueled ramjet combustors, was reported[4] in 1976, is referred to as WL76. The two programs used slightly different combustor designs and utilized different thrust stands for measurements. For evaluation of scaling methods, all of the results can be considered as a single data set as long as no cross comparisons (WL74–WL76) are made.

Both experimental studies at WL used a coaxial flow, single dump station liquid-fueled ramjet. A schematic of the general combustor design is presented in Fig. 2. The ramjet fuel used in both experimental studies was JP4, which was injected transversely into the vitiated airstream through orifices upstream of the dump station. The combustors were fabricated from schedule 40 stainless steel pipe sections. The air inlet duct was fabricated from schedule 40 stainless steel pipe with diameters that equaled one-half of the combustor diameters. A more detailed description of the hardware used in the experimental programs is presented in Refs. 3 and 4.

The goal of the first experimental program was to identify the effects that varying geometry and other operating conditions have on thermal efficiency. This was accomplished by evaluating baseline conditions at several equivalence ratios and then varying the airflow rate, L/D, L^*, A_4/A_5, and the number of fuel injection orifices. These effects were observed in different-sized combustors. The baseline conditions for the different-sized combustors were planned on the basis of *pD* scaling. The conditions that deviate from the baseline conditions include tests that match *mA* scaling and additional tests that match *pD* scaling.

The goal of the second experimental program was to observe the effects of varying the same parameters in larger sized ramjet combustors at lower pressures. In this program, baseline conditions at different equivalence ratios were tested in the different-sized combustors. The effects from varying the other parameters were observed in only one combustor size.

Experimental Results

Data from the WL programs for tests that can be used to evaluate scaling methods are presented. The test conditions are grouped

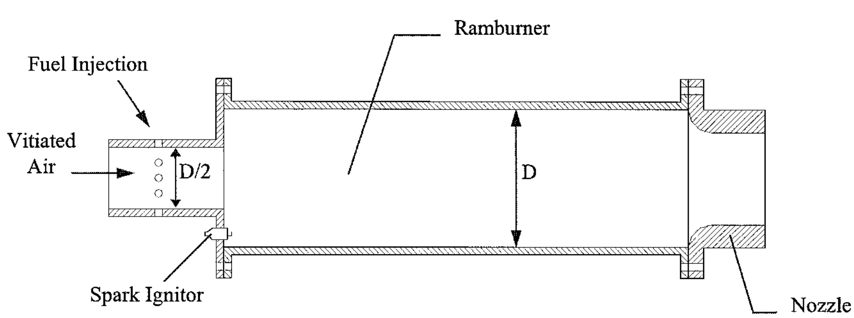

Fig. 2 General design of WL coaxial liquid fuel ramjet.

according to the scaling methods that are matched. The conditions for pD and mA scaling reported in WL74 are presented in Tables 1 and 2, respectively. The tests in WL76 that match pD scaling are presented in Table 3. The test numbers in Tables 1–3 correspond to the order of data reported in Refs. 3 and 4 and are used to provide identification between test conditions and results. The majority of the 2-in. combustor tests reported in Ref. 3 did not accomplish sustained combustion and are not considered for the purpose of evaluating scaling methods. It was noted that heat loss effects may have contributed to the poor performance of the 2-in. combustors.[3]

In Tables 1 and 3, the value for the product of the ideal combustor pressure and the combustor diameter (pD) is reported for each of the pD scaled conditions. Similarly, in Table 2 the ideal combustor pressure is reported for each of the mA scaled conditions. The ideal combustor pressure was determined as described in Ref. 12.

Some WL76 tests were reported as pD scaling tests do not actually match the pD scaling method. In WL76, the 5-in. combustor tests have a constant value for the product of the ideal combustor pressure and the combustor diameter (pD). However, the tests for the 8-, and 12-in. combustors have pD values that increase with increasing fuel-to-air ratio. Therefore, the tests listed in Table 3 do not include all of the tests reported[4] as pD scaling tests in WL76 because they do not meet the criteria for pD scaling.

Table 1 WL74 *pD* scaling test conditions

Test number 5 in.	4 in.	3 in.	Test description	Fuel/air ratio	pD Value (ideal), psia-in.
1	17	38	Baseline	0.026	210
2	18	39	Baseline	0.040	210
3	19	40	Baseline	0.055	210
123	17	38	Baseline	0.025	210
125	18	39	Baseline	0.040	210
127	19	40	Baseline	0.055	210
76	107	——	$L/D = 2.25$	0.025	210
77	108	——	$L/D = 2.25$	0.040	210
78	109	——	$L/D = 2.25$	0.055	210
147	120	——	4 Injection orifices	0.040	210
148	122	——	4 Injection orifices	0.054	210
7	28	——	High pressure	0.055	405
——	23	44	High pressure	0.025	300
——	25	46	High pressure	0.039	300
——	25	46	High pressure	0.054	300
12	33	51	High velocity	0.039	190
13	34	52	High velocity	0.055	190
11	——	50	High velocity	0.025	190
——	29	47	Low velocity	0.026	240
——	30	48	Low velocity	0.039	240
——	31	49	Low velocity	0.055	240

Table 2 WL74 *mA* scaling test conditions

Test number 5 in.	4 in.	3 in.	Test description	Fuel/air ratio	p_4 (ideal), psia
1	20	——	5 in. baseline/4 in. low pressure	0.025	42
2	21	——	5 in. baseline/4 in. low pressure	0.039	42
3	22	——	5 in. baseline/4 in. low pressure	0.054	42
4	23	——	5 in. high pressure/4 in. high pressure	0.025	72
5	24	——	5 in. high pressure/4 in. high pressure	0.040	72
6	25	——	5 in. high pressure/4 in. high pressure	0.055	72
——	17	41	4 in. baseline/3 in. low pressure	0.025	52
——	18	42	4 in. baseline/3 in. low pressure	0.040	52
——	19	43	4 in. baseline/3 in. low pressure	0.055	52
——	26	44	4 in. high pressure/3 in. high pressure	0.025	105
——	27	45	4 in. high pressure/3 in. high pressure	0.040	105
——	28	46	4 in. high pressure/3 in. high pressure	0.055	105

Table 3 WL76 *pD* scaling test conditions

Test number 5 in.	4 in.	3 in.	Test description	Fuel/air ratio	pD Value (ideal), psia-in.
——	206	214	Scaling	0.025	185
——	207	215	Scaling	0.031	200
202	208	216	Scaling	0.033	210
——	209	217	Scaling	0.040	220
——	210	218	Scaling	0.046	230
——	211	219	Scaling	0.051	235
——	212	220	Scaling	0.055	240
——	213	221	Scaling	0.062	250

Table 4 Reported fuel to air ratios related to equivalence ratio

Fuel/air ratio	Equivalence ratio
0.025	0.30
0.040	0.44
0.055	0.58
0.062	0.65

The selected data presented in Tables 1–3 were chosen on the basis of matching the methods of application for the scaling methods within specific bounds. The criteria for deciding whether the conditions in two tests sufficiently correspond to a scaling method were to choose only test conditions whose fuel-to-air ratios were reported to be valued within 0.003 of each other and whose airflow rates were scaled matching either pD or mA scaling.

For the test conditions presented in Tables 1 and 2, there were target fuel-to-air ratios of 0.025, 0.040, and 0.055, whereas the test conditions in Table 3 covered a slightly broader range of values. Reported fuel-to-air ratio values are related to the equivalence ratios for the vitiated air/JP4 mixture as presented in Table 4. The reported fuel-to-air ratio does not include the vitiator fuel and oxygen contribution, which is estimated as discussed later. In Table 4 the range of fuel-to-air ratios represent the range used in the two test programs. It should be noted that the equivalence-ratio range evaluated in the test conditions represents typical operating flight conditions for a ramjet. Consequently, the equivalence ratios evaluated in WL74 and WL76 do not approach the fuel-lean and fuel-rich flame-out limits.

The thermal efficiency values reported in the WL technical reports are based on thrust measurements. Which of the eight possible thrust-based data-reduction methods[12,13] used to determine thermal efficiency is unknown. Also, between the two test programs the thrust stand for the test facility was replaced due to the age of the stand. In the first experimental program, pressure measurements were recorded and, therefore, thermal efficiency can be estimated independent of thrust for comparison and validation of the reported values.

To accurately estimate the thermal efficiency based on pressure measurements, the vitiator flow rates must be known or approximated. In discussions with personnel at WL, it was reported that JP4 was used as the vitiator fuel and that replenishment oxygen was added so that the oxygen mass percentage of the vitiator products was consistent with ambient air. Also, the heater was iteratively designed and evaluated to perform at a nominal value of 95% thermal efficiency.

The mass percentage method of oxygen replenishment requires the following chemical equation for the vitiator:

$$C_9H_{17} + 18.1 \cdot O_2 + \text{air} = 8.5 \cdot H_2O + 9 \cdot CO_2 + 4.8 \cdot O_2 + \text{air} \tag{15}$$

where JP4 is expressed as C_9H_{17}. Equation (15) establishes the proper ratio of JP4 to replenishment oxygen to estimate the vitiator flow rates. For the analysis, the flow rates were estimated assuming 95% heater thermal efficiency based on chemical equilibrium at the reported inlet temperature using the JANNAF PEP code.[14] The procedure for estimating vitiator flow rates is presented in Ref. 12. Note that the reported airflow rate provided in the WL technical reports[3,4] is the airflow rate without the vitiator fuel and oxygen.

Figure 3 shows the calculated pressure based thermal efficiency estimates using the method described as "method (1,2)" in Refs. 12 and 13 compared to the reported thrust-based values for the WL74 5-in. combustor tests used in the *pD* scaling and *mA* scaling analysis. As can be seen in Fig. 3, the pressure-based estimates of the thermal efficiency are very similar to the reported thrust-based thermal efficiency values. The 4- and 3-in. combustor tests used for the scaling investigation also showed that pressure-based thermal efficiency estimates have similar values compared to the reported thrust-based values. The estimates of the thermal efficiency based on the pressure measurements help to validate the reported thrust-based values. Because the pressure-based estimates are made using estimates of the vitiator flow rates, the reported thrust-based values are used for the analysis of the scaling methods.

Because sufficient data were reported,[3] C^* efficiency estimates were made using the method described as "method 2" in Refs. 12 and 13. Figure 4 shows pairs of $\eta_{\Delta T}$ and η_{C^*} estimates based on pressure for the 5-in. combustor tests and arranged in order of increasing $\eta_{\Delta T}$. The values in Fig. 4 support the discussion in Refs. 12 and 13 regarding the limited resolution of values for η_{C^*} compared to $\eta_{\Delta T}$. The trends of η_{C^*} in Fig. 4 do not exactly correlate to trends in $\eta_{\Delta T}$. The pairs of values are arranged in order of increasing $\eta_{\Delta T}$. However, η_{C^*} does not always increase from one set of values to another. This is because at different flow rates, the theoretical pressure rise in the combustor is different, which results in different resolution of possible values for η_{C^*}. Figure 4 shows why η_{C^*} should not be used for ramjet performance determination (as discussed in Refs. 12 and 13).

Although no effort to evaluate the repeatability of performance at identical test conditions was recorded, there are some data which can be reviewed as an indication of performance repeatability. The conditions and reported efficiencies for the similar tests are presented in Table 5. The maximum difference in thermal efficiency for similar test conditions is 5.7%. The average difference for the 10 comparable sets of thermal efficiency values is 2.6%. The precision uncertainty in the WL tests is not known, but it appears from the values in Table 5 that an uncertainty estimate of 3–4% of reading, is of the correct order.

Analysis of Results

To objectively evaluate the success of the scaling methods, a comparison of thermal efficiency values at equivalent-scaled conditions for the different-sized combustors is used. In comparing the values, it is necessary to consider the overall uncertainties in the thermal efficiencies to determine if a match is achieved. In this analysis, the words *match* and *matching* are used to denote cases that may have a common true value for thermal efficiency for two different-sized combustors. Three different levels of estimated uncertainty for the thermal efficiencies are used so that the matching of the scaling methods can be observed as a function of possible uncertainty level.

The uncertainty in a result is given by[15]

$$U_r = \left[(B_r)^2 + (P_r)^2\right]^{\frac{1}{2}} \tag{16}$$

where B_r is the bias or systematic uncertainty, P_r the precision uncertainty, and U_r the overall uncertainty in the result. Typical bias uncertainty estimates for the thrust-based thermal efficiency

Table 5 Repeatability data for WL74 test program

Test condition	Test number	Combustor diameter	Fuel/air ratio	$\eta_{\Delta T}$, %
Baseline	1	5	0.027	62.3
Baseline	123	5	0.028	68.0
4 Injection orifices	146	5	0.028	63.1
Baseline	2	5	0.039	61.8
Baseline	125	5	0.040	66.4
4 Injection orifices	147	5	0.041	65.6
Baseline	3	5	0.054	74.9
Baseline	127	5	0.053	74.4
$L/D = 6$	82	5	0.025	89.1
$L/D = 6$	131	5	0.027	88.4
$L/D = 6$	83	5	0.040	91.5
$L/D = 6$	133	5	0.042	89.6
$L/D = 6$	84	5	0.055	86.4
$L/D = 6$	135	5	0.055	88.4

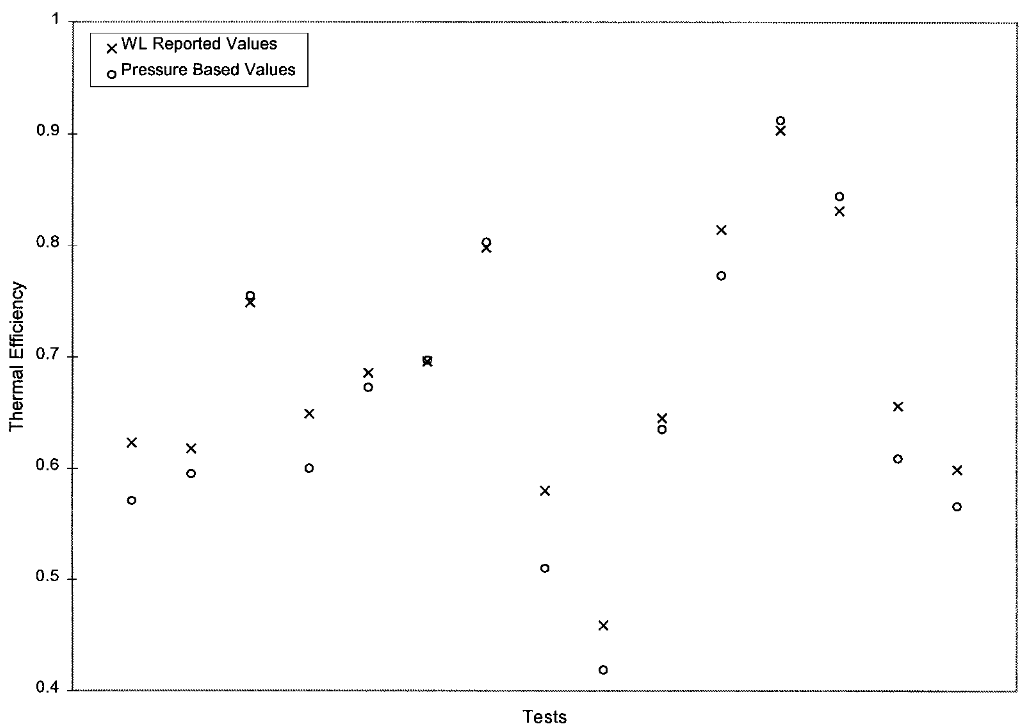

Fig. 3 WL reported vs pressure-based thermal efficiency for 5-in. combustor tests.

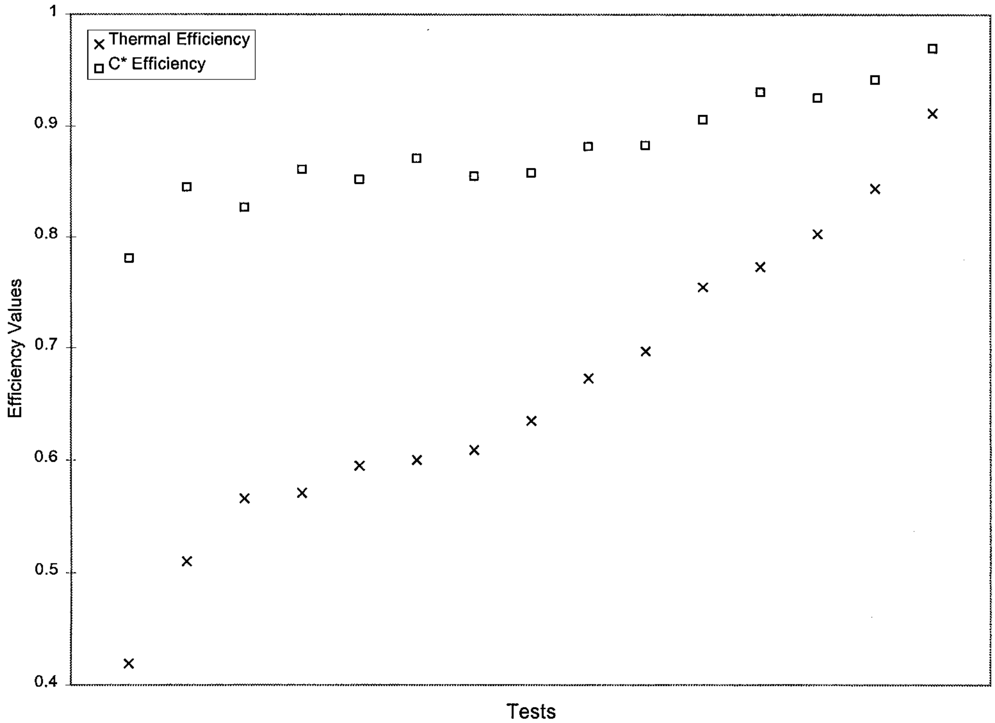

Fig. 4 Thermal efficiency and C^* efficiency values.

methods are presented in Refs. 12 and 13 and range from 2.9 to 4.1% of reading for the eight different thrust-based methods. A bias uncertainty estimate of 3.5% of reading was chosen for this study. Thus, the precision uncertainty and the bias uncertainty are estimated to be approximately equal. Three different estimates of the overall result uncertainty in the thermal efficiency values are used in evaluating the scaling methods. The values of 3.5% of reading, 5% of reading, and 8% of reading correspond to cases with typical precision uncertainty only, combined bias and precision uncertainties, and an upper bound uncertainty, respectively.

When checking the criteria for considering matches between different-sized combustor tests, one might consider that any overlapping of uncertainty bands would be sufficient to indicate a match for the specified uncertainty estimate. This would not be the most appropriate approach. Recall that the uncertainty estimates are based on a 95% coverage of the true value within the uncertainty band. If all overlapping uncertainty bands were considered matches, there may be cases where the overlap is very close to the outer limits of both uncertainties, which would result in conditions that counted as matches that are not statistically likely to contain a true value of thermal efficiency that satisfies both tests. A more appropriate approach was presented by Coleman and Stern[16] in considering validation of computational fluid dynamics (CFD) codes with experimental data. Following a similar approach, consider the equation

$$\Delta = |\eta_{\Delta T,1} - \eta_{\Delta T,2}| \tag{17}$$

where Δ is the difference between two thermal efficiency values represented as $\eta_{\Delta T,1}$ and $\eta_{\Delta T,2}$. This difference, Δ, would be zero for an exact scaling match with perfect measurements. Then, the uncertainty U_Δ in the difference of the thermal efficiency values can be expressed as

$$U_\Delta^2 = \left(\frac{\partial \Delta}{\partial \eta_{\Delta T,1}}\right)^2 U_{\eta_{\Delta T,1}}^2 + \left(\frac{\partial \Delta}{\partial \eta_{\Delta T,2}}\right)^2 U_{\eta_{\Delta T,2}}^2 = U_{\eta_{\Delta T,1}}^2 + U_{\eta_{\Delta T,2}}^2 \tag{18}$$

where $U_{\eta_{\Delta T,1}}$ and $U_{\eta_{\Delta T,2}}$ represent the uncertainties in the thermal efficiency values that are being compared. When Δ is less than its uncertainty U_Δ, the two compared tests are considered a match for the scaling method being considered.

Note that Eq. (18) does not include the effects of correlated bias uncertainties[17] in the comparison of the thermal efficiency values. Considering that the compared tests were conducted in the same facility, the correlated bias effects would decrease the uncertainty U_Δ to a value approximated by using only the precision uncertainty values of the thermal efficiencies in Eq. (18). Thus, the $U_{\eta_{\Delta T}} = 3.5\%$ case appears to be the most appropriate for evaluating the scaling methods.

One can observe that an increase in the uncertainty estimates of the thermal efficiency values will increase the number of matches. However, as the uncertainty in the data increases, a point is reached where the data become useless from a practical standpoint.

Tables 6 and 7 show the test data and number of matches for the pD and mA scaling methods for tests reported in WL74 for the different uncertainty estimates. Table 8 presents the data and matches for the tests in WL76. The tests that did not sustain combustion are also listed in these tables, but are not compared for matches.

The comparisons for tests of pD and mA scaling from WL74 represent a much broader study than those tests from WL76, which consider pD scaling for larger size combustors at varying equivalence ratios. A breakdown of the different parameters that varied in WL74 tests is provided in Table 9, where a fractional value is given to represent the number of matches over the number of test conditions where applicable.

Discussion of Results

The analysis of results indicates that the two scaling methods provide matching of thermal efficiency for different-sized combustors only in a limited number of test conditions. Both pD and mA scaling consistently failed to achieve a high rate of matched thermal efficiencies for all of the different conditions (WL74) presented in Tables 6 and 7, which are further analyzed in Table 9.

The experimental results that did show better matching were the limited number of tests (WL76) of the 5-, 8-, and 12-in. combustors that used pD scaling and that are presented in Table 8. In these tests, the ideal pressures of the combustors were lower than those of the WL74 scaling tests due to the larger size combustors and the method of application for pD scaling. It is not known if the increased matches

Table 6 WL74 tests: *pD* scaling analysis

Test numbers			5 in.	4 in.	3 in.		U_Δ for the case $U_{\eta\Delta T} = 3.5\%$ of reading, %	U_Δ for the case $U_{\eta\Delta T} = 5.0\%$ of reading, %	U_Δ for the case $U_{\eta\Delta T} = 8.0\%$ of reading, %
5 in.	4 in.	3 in.	$\eta_{\Delta T}$, %	$\eta_{\Delta T}$, %	$\eta_{\Delta T}$, %	Δ, %			
1	17	—	62.3	71.4	—	9.1	3.3	4.7	7.6
1	—	38	62.3	—	62.3	0.0	3.1√[a]	4.4√	7.0√
—	17	38	—	71.4	62.3	9.1	3.3	4.7	7.6
2	18	—	61.8	83.6	—	21.8	3.6	5.2	8.3
2	—	39	61.8	—	74.9	13.1	3.4	4.9	7.8
—	18	39	—	83.6	74.9	8.7	3.9	5.6	9.0√
3	19	—	74.9	81.2	—	6.3	3.9	5.5	8.8√
3	—	40	74.9	—	72.5	2.4	3.6√	5.2√	8.3√
—	19	40	—	81.2	72.5	8.7	3.8	5.4	8.7√
123	17	—	68.0	71.4	—	3.4	3.5√	4.9√	7.9√
123	—	38	68.0	—	62.3	5.7	3.2	4.6	7.4√
125	18	—	66.4	83.6	—	17.2	3.7	5.3	8.5
125	—	39	66.4	—	74.9	8.5	3.5	5.0	8.0
127	19	—	74.4	81.2	—	6.8	3.9	5.5	8.8√
127	—	40	74.4	—	72.5	1.9	3.6√	5.2√	8.3√
76	107	—	58.0	52.5	—	5.5	2.7	3.9	6.3√
77	108	—	45.9	48.9	—	3.0	2.3	3.4√	5.4√
78	109	—	64.5	69.9	—	5.4	3.3	4.8	7.6√
147	120	—	65.6	63.6	—	2.0	3.2√	4.6√	7.3√
148	122	—	59.9	75.9	—	16.0	3.4	4.8	7.7
7	28	—	79.8	81.0	—	1.2	4.0√	5.7√	9.1√
—	23	44	—	68.1	63.0	5.1	3.2	4.6	7.4
—	24	45	—	84.3	77.1	7.2	4.0	5.7	9.1√
—	25	46	—	83.3	76.1	7.2	3.9	5.6	9.0√
12	33	—	64.8	70.8	—	6.0	3.4	4.8	7.7√
12	—	51	64.8	—	70.9	6.1	3.4	4.8	7.7√
—	33	51	—	70.8	70.9	0.1	3.5√	5.0√	8.0√
11	—	50	63.9	—	56.3	7.6	3.0	4.3	6.8
—	29	47	—	64.5	70.0	5.5	3.3	4.8	7.6√
—	30	48	—	74.6	65.5	9.1	3.5	5.0	7.9
13	34	—	54.2	*[b]	—	—	—	—	—
13	—	52	54.2	—	*	—	—	—	—
—	34	52	—	*	*	—	—	—	—
—	31	49	—	66.7	*	—	—	—	—
Number of matching cases ($\Delta < U_\Delta$) for 30 comparisons							7	8	21
% Matching of total comparisons							23	27	70

[a] √ Indicates a match. [b] *Test did not sustain combustion.

Table 7 WL74 tests: *mA* scaling analysis

Test numbers			5 in.	4 in.	3 in.		U_Δ for the case $U_{\eta\Delta T} = 3.5\%$ of reading, %	U_Δ for the case $U_{\eta\Delta T} = 5.0\%$ of reading, %	U_Δ for the case $U_{\eta\Delta T} = 8.0\%$ of reading, %
5 in.	4 in.	3 in.	$\eta_{\Delta T}$, %	$\eta_{\Delta T}$, %	$\eta_{\Delta T}$, %	Δ, %			
1	20	—	62.3	63.7	—	1.4	3.1√[a]	4.5√	7.1√
2	21	—	61.8	76.5	—	14.7	3.4	4.9	7.9
3	22	—	74.9	75.9	—	1.0	3.7√	5.3√	8.5√
125	21	—	66.4	76.5	—	10.1	3.5	5.1	8.1
127	22	—	74.4	75.9	—	1.5	3.7√	5.3√	8.5√
—	17	41	—	71.4	66.7	4.7	3.4	4.9√	7.8√
—	18	42	—	83.6	*[b]	—	—	—	—
—	19	43	—	81.2	*	—	—	—	—
4	23	—	64.9	68.1	—	3.2	3.3√	4.7√	7.5√
5	24	—	68.6	84.3	—	15.7	3.8	5.4	8.7
6	25	—	69.6	83.3	—	13.7	3.8	5.4	8.7
—	26	44	—	67.1	63.0	4.1	3.2	4.6√	7.4√
—	27	45	—	83.8	77.1	6.7	4.0	5.7	9.1√
—	28	46	—	81.0	76.1	4.9	3.9	5.6√	8.9√
Number of matching cases ($\Delta < U_\Delta$) for 12 comparisons							4	7	8
% Matching of total comparisons							33	58	67

[a] √ Indicates a match. [b] *Test did not sustain combustion.

for *pD* scaling is due to the lower combustion pressures or possibly some other reason, such as heat loss having less effect on the larger size combustors. However, for the 3.5% of reading uncertainty level, which may best characterize the comparison of the WL test results, only 6 of 10 tests were matched. Also, it is important to note that the WL76 tests represent a much less comprehensive set of test conditions and parameter changes than do the WL74 tests.

The overall results of Table 9 indicate that the scaling methods for the WL74 3-, 4-, and 5-in. combustor tests do not match over the range of variations in test conditions for either of the scaling methods. The tests that used *pD* scaling have a wider range of different parameters including different combustor Mach number, combustion pressure, combustor length-to-diameter ratios and fuel-to-air ratios. The tests that correspond to *mA* scaling include changes in combustion pressure and fuel-to-air ratio. Altogether, even at the uncertainty level of 5% of reading, the percentage of matched conditions was 27 and 58% for *pD* and *mA* scaling, respectively. This may lead one to conclude that *mA* scaling is superior to *pD* scaling for the conditions evaluated. However, the goal of any scaling method is to scale consistently the thermal efficiency values from one size

Table 8 WL76 tests: *pD* scaling analysis

Test numbers 5 in.	Test numbers 8 in.	Test numbers 12 in.	5 in. $\eta_{\Delta T}$, %	8 in. $\eta_{\Delta T}$, %	12 in. $\eta_{\Delta T}$, %	Δ, %	U_Δ for the case $U_{\eta\Delta T}=3.5\%$ of reading, %	U_Δ for the case $U_{\eta\Delta T}=5.0\%$ of reading, %	U_Δ for the case $U_{\eta\Delta T}=8.0\%$ of reading, %
——	206	214	——	75.6	75.7	0.1	3.7√[a]	5.3√	8.5√
——	207	215	——	81.8	79.3	2.5	4.0√	5.7√	9.1√
202	208	——	75.6	76.4	——	0.8	3.8√	5.4√	8.6√
202	——	216	75.6	——	77.5	1.9	3.8√	5.4√	8.7√
——	208	216	——	76.4	77.5	1.1	3.8√	5.4√	8.7√
——	209	217	——	77.7	83.3	5.6	4.0	5.7√	9.1√
——	210	218	——	84.9	80.4	4.5	4.1	5.8√	9.4√
——	211	219	——	83.5	80.7	2.8	4.1√	5.8√	9.3√
——	212	220	——	84.3	80.1	4.2	4.1	5.8√	9.3√
——	213	221	——	84.0	78.0	6.0	4.0	5.7	9.2√
Number of matching cases ($\Delta < U_\Delta$) for 10 comparisons							6	9	10
% Matching of total comparisons							60	90	100

[a]√ Indicates a match.

Table 9 Comparison of scaling methods for various conditions

Test conditions	Matches for $U_{\eta\Delta T}=3.5\%$ of reading: *mA*	*pD*
Theoretical combustor pressure (5 in. pressure for pD scaling)		
40–60 psia	$\frac{3}{6}$	$\frac{6}{26}$
60–80 psia	$\frac{1}{3}$	$\frac{0}{3}$
>80 psia	$\frac{0}{3}$	$\frac{1}{1}$
Combustor Mach number		
Mcomb $= 0.1$	——	$\frac{0}{2}$
Mcomb $= 0.3$	$\frac{4}{12}$	$\frac{6}{24}$
Mcomb $= 0.5$	——	$\frac{1}{4}$
Length/diameter		
$L/D = 3.0$	$\frac{4}{12}$	$\frac{7}{27}$
$L/D = 2.25$	——	$\frac{0}{3}$
Target fuel/air ratio		
0.025	$\frac{2}{4}$	$\frac{2}{9}$
0.040	$\frac{0}{4}$	$\frac{2}{12}$
0.055	$\frac{2}{4}$	$\frac{3}{9}$

combustor to another. In such a case, if only 50–60% of the tests provide correctly scaled values, there would be no confidence in the scaled results.

Implications of Results of Scaling Tests

From the data presented for both *pD* and *mA* scaling, there is no convincing evidence to support the use of either method for predicting performance of one combustor by testing a combustor of another size. In fact, the experimental data support the conclusion that both methods fail to provide appropriate scaling of results.

If one must choose one of these methods for scaling, however, there are other implications and observations that can be made. The *pD* scaling method is more difficult to implement in a subscale test program. The air requirement (and, consequently, vitiator flow requirement) for scaling down in size using *pD* scaling is more than that of *mA* scaling. For instance, if a subscale combustor has a diameter of one-half of the full-scale combustor, *pD* scaling requires one-half of the full-scale flow rate, whereas *mA* scaling requires only one-quarter of the full-scale flow rate. This difference in flow rate requirement allows a greater scaling range for the least facility requirements when using *mA* scaling. Furthermore, the higher operational pressure of *pD* scaling requires more robust hardware, which may incur additional costs.

Another observation that can be made from the analytical discussion of similarity in ramjet combustion is that *mA* scaling may be more appropriate when scaling combustors that do not rely strictly on global fluid mechanics of the flowfield for flame holding. In some ramjets, screens to promote fine-scale turbulence or other geometric obstructions are used to promote flame holding. In such cases, *mA* scaling may well be the better method because the chemical kinetics would still be matched and the flowfield replication may not contribute as much to performance. Another similar case occurs when hot particles from a gas generator are exhausted into the ramjet burner to promote ignition and flame holding. In that case, localized chemical kinetics would appear to be more important than flowfield replication, which again would imply that *mA* scaling might be the better scaling method for that case.

Conclusions

The evaluation of scaling methods indicates that neither *pD* or *mA* scaling provides satisfactory scaling of thermal efficiency in the different-sized combustors. Despite previous studies[3–5] that endorse the use of *pD* scaling, the only experimental evidence available indicates that the method does not provide consistent scaling of results throughout the range of experimental conditions that have been reported if reasonable levels of experimental uncertainty are assumed.

However, if one must use one of the scaling methods, there are several reasons to choose *mA* scaling. The method is easier to implement, provides a greater scaling range (for a given facility), and may match conditions that do not require flame holding by large-scale fluid mechanic structures.

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